Parallel Concatenated Trellis Coded Modulation[1]

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ABSTRACT: In this paper, we propose a new solution to parallel concatenation of trellis codes with multilevel amplitude/phase modulations and a suitable bit by bit iterative decoding structure. Examples are given for throughput 2 and 4 bits/sec/Hz with 8PSK, 16QAM, and 64QAM modulations. For parallel concatenated trellis codes in the examples, rate 2/3 and 4/5, 8, and 16-state binary convolutional codes with Ungerboeck mapping by set partitioning (natural mapping), a reordered mapping, and Gray code mapping are used. The performance of these codes is within 1 dB from the Shannon limit at a bit error probability of 10^{-7} for a given throughput, which outperforms the performance of all codes reported in the past for the same throughput.

I. Introduction

Trellis coded modulation (TCM) proposed by Ungerboeck in 1982 [1] is now a well-established technique in digital communications. Since its first appearance, TCM has generated a continually growing interest, concerning its theoretical foundations as well as its numerous applications, spanning high-rate digital transmission over voice circuits, digital microwave radio relay links, and satellite communications. In essence, it is a technique to obtain significant coding gains (3-6 dB) sacrificing neither data rate nor bandwidth.

Turbo codes represent a more recent development in the coding research field [2], which has risen a large interest in the coding community. They are *parallel concatenated convolutional codes* (PCCC) whose encoder is formed by two (or more) *constituent* systematic encoders joined through one or more interleavers. The input information bits feed the first encoder and, after having been scrambled by the interleaver, they enter the second encoder. A codeword of a parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders.

The suboptimal[2] iterative decoding structure is modular, and consists of a set of concatenated decoding modules, one for each constituent code, connected through the same interleavers used at the encoder side. Each decoder performs weighted soft decoding of the input sequence. Bit error probabilities as low as 10^{-6} at $E_b/N_0 = -0.6$ dB have been shown by simulation [4] using codes with rates as low as 1/15. Parallel concatenated convolutional codes yield very large coding gains (10-11 dB) at the expense of a data rate reduction, or bandwidth increase.

It seems thus worthwhile to merge TCM and PCCC in order to obtain large coding gains and high bandwidth efficiency. A first attempt employing the so-called "pragmatic" approach to TCM was described in [5]. Later, turbo codes were embedded in multilevel codes with multistage decoding [7]. Recently [8], punctured versions of Ungerboeck codes were used to construct turbo codes for 8PSK modulation. In this paper, we propose a new solution to parallel concatenation of trellis coded modulation (PCTCM) with multilevel amplitude/phase modulations and a suitable bit-by-bit iterative decoding structure. The proposed PCTCM are analyzed using both simulation and an analytical technique based on that described in [3] [9] and [4]. The performance of the new codes is within 1 dB from the Shannon limit at bit error probabilities of 10^{-7}, and outperform all codes reported previously for the same throughput.

II. Parallel Concatenated Trellis Coded Modulation

Various approaches for turbo codes with multilevel modulation were proposed in [5] [7] and [8]. Here we propose a different approach that outperforms the results in [5] [7] and [8] when M-QAM or MPSK modulation is used, in particular at low bit error rates, less than 10^{-6}. A straightforward method to use parallel concatenated codes with multilevel modulation is first to select a rate $\frac{b}{b+1}$ constituent code where the outputs are mapped to a 2^{b+1}-level modulation based on Ungerboeck's set partitioning method (i.e., we can use Ungerboeck's codes with feedback). If MPSK modulation is used, for every b bits at the input of the parallel concatenated encoder we transmit two consecutive 2^{b+1} PSK signals, one per each encoder output. This results in a throughput of $b/2$ bits/sec/Hz. If M-QAM modulation is used, we map the $b+1$ outputs of the first component code to the 2^{b+1} in-phase levels (I-channel) of a 2^{2b+2}-QAM signal set, and the $b+1$ outputs of the second component code to the 2^{b+1} quadrature levels (Q-channel). The throughput of this system is b bits/sec/Hz.

We note that these methods require more levels of modulation than conventional TCM, which is not desirable in practice. Moreover, the input information sequences are used twice in the output modulation symbols, which is also not desirable. In contrast, turbo codes for binary modulation transmit the uncoded information only once.

An obvious remedy would be to puncture the output symbols of each trellis code and select the puncturing pattern such that the output symbols of the parallel concatenated code contain the input information only once. If the output symbols of the first encoder is punctured uniformly, the puncturing pattern of the second trellis code is non-uniform and depends on the particular choice of interleaver. In this way, for example, for 2^{b+1}-PSK a throughput b can be achieved. This method was proposed in [8]. The method uses symbol interleaving, and the reliability of punctured symbols may not be reproducible at the decoder.

A New Solution to Parallel Concatenated TCM — A better remedy to obtain a rate $\frac{b}{b+1}$ (b even) constituent code, is to select $b/2$ systematic outputs and puncture the rest of the systematic outputs, but use the parity bit of the $\frac{b}{b+1}$ code (Note that the constituent code of rate $\frac{b}{b+1}$ may have been already derived by puncturing a rate 1/2 code). Then do the same to the second constituent code, but select only those systematic bits which were punctured in the first encoder.

This method requires at least two interleavers: the first interleaver permutes the bits selected by the first encoder and the second interleaver those punctured by the first encoder. For MPSK (or MQAM) we can use $2^{1+b/2}$ PSK symbols (or $2^{1+b/2}$ QAM symbols) per encoder and achieve throughput $b/2$. For M-QAM we can also use $2^{1+b/2}$ levels in the I-channel and $2^{1+b/2}$ levels in the Q-channel, and achieve a throughput of b bits/sec/Hz.

These methods are equivalent to a multi-dimensional trellis coded modulation scheme (in this case, two multi-level symbols per branch) which uses $2^{b/2} \times 2^{1+b/2}$ signal points, where the first symbol in the branch (which only depends on uncoded information) is punctured. Now, with

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[2] Although not formally proved, the suboptimum algorithm yields performance very close to the maximum-likelihood algorithm [3].

Signal levels	0	1	2	3
Natural mapping	00	01	10	11
Reordered mapping	00	01	10	11
Gray code mapping	00	01	11	10

Table 1: Mappings for each dimension of 16QAM.

Signal levels or Cosets	0	1	2	3	4	5	6	7
Natural mapping	000	001	010	011	100	101	110	111
Reordered mapping	000	001	010	011	110	111	100	101
Gray code mapping	000	001	011	010	110	111	101	100

Table 2: Mappings for 8PSK and each dimension of 64QAM.

these methods the reliability of the punctured symbols is reproducible at the decoder.

To optimize the PCTCM code, the constituent codes for a given modulation should be designed based on the Euclidean distance.

Design and Selection of Parallel Concatenated TCM— The design criterion for turbo codes with binary modulation is discussed in [9] and [4]. To achieve very low bit error rates, one should maximize the effective free distance of turbo code [9] [4]. In order to select parallel concatenated TCM schemes using random interleavers we extend this criterion to nonbinary modulation.

Let $\mathbf{u}$ be the transmitted binary information sequence, and $x(\mathbf{u})$ be the corresponding turbo encoder output with M-ary symbols.

The criteria to design and select constituent TCM codes are:

1. Effective free Euclidean distance — Choose the constituent TCM encoders with a given mapping (binary labels for cosets or signal levels) such that the minimum Euclidean distance $d(x(\mathbf{u}), x(\mathbf{u}'))$ over all $\mathbf{u}, \mathbf{u}'$ pairs such that $\mathbf{u} \neq \mathbf{u}'$ is maximized, given that the Hamming distance $d_H(\mathbf{u}, \mathbf{u}') = 2$. We call this minimum Euclidean distance the *effective free Euclidean distance* of parallel concatenated TCM and denote it simply by d_{ef}.

2. Mapping — In this paper we use three types of mapping: Ungerboeck mapping by set partitioning (natural mapping), reordered mapping, and Gray code mapping. In Tables 1 and 2 signal levels or cosets and the corresponding binary labels are shown for these three mappings. The reordered mapping was used in [10] for other reasons. To better understand the reordered mapping, consider an 8PSK constellation which has eight cosets $c_0, c_1, .., c_7$. Partition the cosets into two groups c_0, c_2, c_4, c_6 and c_1, c_3, c_5, c_7. (In the binary labels of the cosets, LSB=0 represents the first group and LSB=1 represents the second group). Swap the last two cosets in each group to obtain the groups c_0, c_2, c_6, c_4 and c_1, c_3, c_7, c_5. Then recompose the eight cosets into the reordered cosets $c_0, c_1, c_2, c_3, c_6, c_7, c_4, c_5$. For example if b_2, b_1, b_0 represents a binary label for natural mapping, where b_2 is the MSB and b_0 is the LSB, then the reordered mapping is given by b_2, (b_2+b_1), b_0. For Gray code mapping we have b_2, (b_2+b_1), (b_1+b_0). Note that the reordered mapping for 4-level signals is the same as natural mapping.

3. Structure of encoders — The canonical structure of TCM encoders using systematic recursive $b/(b+1)$ convolutional codes is shown in Fig. 1 . For block by block encoding a trellis termination method as discussed in [4] is also shown in the same Figure.

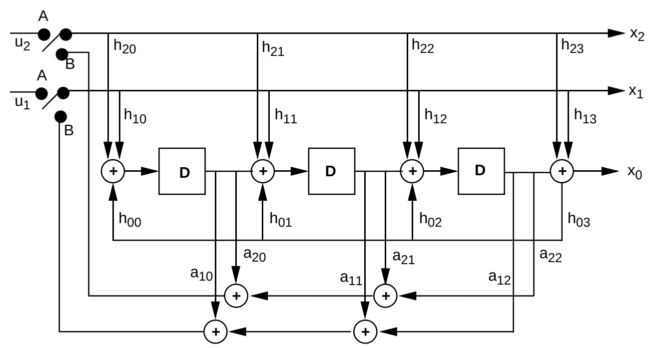

Figure 1: Canonical structure of rate $b/(b+1)$ encoder. ($b = 2$, $m = 3$)

4. Splitting interleavers to obtain larger d_{ef} — Here we propose a method for possible improvement of d_{ef}, when $b > 2$. As we mentioned in the previous section, for two parallel concatenated TCM we should use at least two interleavers. Sometimes it is possible to improve d_{ef} by using up to $b > 2$ interleavers. The price we pay is a sacrifice in interleaving gain since the size of each interleaver now is decreased, if we want to keep constant the total size of interleavers.

   The input vector $\mathbf{u}$ can be decomposed into k subsequences as $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k)$ where k, $2 \le k \le b$, represents the number of interleavers used. Here k is a multiple of number of encoders used, and for two codes k is even. Thus we have $d_H(\mathbf{u}, \mathbf{u}') = d_H(\mathbf{u}_1, \mathbf{u}_1') + d_H(\mathbf{u}_2, \mathbf{u}_2') + \cdots + d_H(\mathbf{u}_k, \mathbf{u}_k')$.

   Also note that $\pi\{\mathbf{u}\} + \pi\{\mathbf{u}'\} = \pi\{\mathbf{u} + \mathbf{u}'\}$, where $\pi\{\cdot\}$ denotes the permutation operation and additions are modulo-2. Thus $d_H(\pi\{\mathbf{u}\}, \pi\{\mathbf{u}'\}) = d_H(\mathbf{u}, \mathbf{u}')$. In our design for simplicity we use identical constituent codes, and interleavers connecting inputs of the two encoders in "reverse order" (see for example Fig. 3).

   Define $w_i = d_H(\mathbf{u}_i, \mathbf{u}_i')$ as the pairwise input weights, and $d_{2,j}(w_1, w_2, \ldots, w_k)$ as the minimum Euclidean distance for pairwise input weights $w_1, w_2, \ldots, w_k$ for encoder j, such that $\sum_1^k w_i = 2$.

   Then we select the codes, such that $d_{ef}^2 = d_{2,1}^2(w_1, w_2, \ldots, w_k) + d_{2,2}^2(w_k, \ldots, w_2, w_1)$ is maximum when $\sum_1^k w_i = 2$.

5. Selection of codes with memory m — Referring to Fig. 1, we select the feedback polynomial $\mathbf{h}_0$ to be primitive.

   For feedforward connections we use the following setups: For natural and reordered mappings we set $h_{i,0} = h_{i,m} = 0$ for $i = 1, \ldots, b/2$, and $h_{i,0} = h_{i,m} = 1$ for $i = (b/2+1), \ldots, b$ in order to maximize the separation between signal points when diverging from a state and when remerging to a state. For Gray mapping we set $h_{i,0} = h_{i,m} = 1$ for $i = 1, \ldots, b$, again for the same reasons.

Based on the above criteria, the best 8 and 16 state codes for 16QAM, 8PSK, and 64QAM were selected, and the corresponding simulation results are reported in Sec. IV.

III. Bit by Bit Iterative Decoding for Parallel Concatenated Trellis Codes

In [4] we described an iterative (turbo) decoding scheme for q parallel concatenated convolutional codes based on approximating the optimum bit decision rule by considering the combination of interleaver and the trellis encoder as a block encoder. The scheme is based on solving a set of nonlinear equations given by ($q = 2$ is used to illustrate the concept)

$$\tilde{L}_{1k} = \log \frac{\sum_{\mathbf{u}:u_k=1} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{2j}}}{\sum_{\mathbf{u}:u_k=0} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{2j}}}$$

$$\tilde{L}_{2k} = \log \frac{\sum_{\mathbf{u}:u_k=1} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{1j}}}{\sum_{\mathbf{u}:u_k=0} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{1j}}} \quad (1)$$

for $k = 1, 2, \cdots, N$. In Eq. (1) $\tilde{L}_{ik}$ are the extrinsic information and $\mathbf{y}_{ik}$ are the received complex observation vectors corresponding to the ith trellis code (see Fig. 2). The final decision is then based on $L_k = \tilde{L}_{1k} + \tilde{L}_{2k}$, which is passed through a hard limiter with zero threshold.

The above set of nonlinear equations are derived from the optimum bit decision rule, i.e.,

$$L_k = \log \frac{\sum_{\mathbf{u}:u_k=1} P(\mathbf{y}_1|\mathbf{u}) P(\mathbf{y}_2|\mathbf{u})}{\sum_{\mathbf{u}:u_k=0} P(\mathbf{y}_1|\mathbf{u}) P(\mathbf{y}_2|\mathbf{u})} \quad (2)$$

using the following approximation

$$P(\mathbf{u}|\mathbf{y}_i) \approx \prod_{k=1}^{N} \frac{e^{u_k \tilde{L}_{ik}}}{1 + e^{\tilde{L}_{ik}}} \quad (3)$$

Note that $P(\mathbf{u}|\mathbf{y}_i)$ is not separable in general. The smaller is the Kullback cross entropy between the right and the left distributions in Eq. (3), the better is the approximation thus the closer may be the iterative decoding to optimum bit decision (This issue has not yet been completely clarified or proven). Instead of using the minimum cross entropy algorithm to convert a non-separable distribution to an approximately separable distribution, we used the MAP algorithm [6] as a non-separable to separable distribution converter, even though such a conversion may not minimize the Kullback cross entropy. We attempted to solve the nonlinear equations in Eq. (1) for $\tilde{\mathbf{L}}_1$, and $\tilde{\mathbf{L}}_2$ by using an iterative procedure

$$\tilde{L}_{1k}^{(m+1)} = \log \frac{\sum_{\mathbf{u}:u_k=1} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{2j}^{(m)}}}{\sum_{\mathbf{u}:u_k=0} P(\mathbf{y}_1|\mathbf{u}) \prod_{j\neq k} e^{u_j \tilde{L}_{2j}^{(m)}}} \quad (4)$$

for $k = 1, 2, \cdots, N$, iterating on m. Similar recursions hold for $\tilde{L}_{2k}^{(m)}$. We start the recursion with the initial condition[3] $\tilde{\mathbf{L}}_1^{(0)} = \tilde{\mathbf{L}}_2^{(0)} = 0$. For the computation of Eq. (4), we use the symbol MAP algorithm [6] with permuters (direct and inverse) where needed, as shown in Fig. 2. The MAP algorithm always starts and ends at the all-zero state since we always terminate the trellis as described in [4].

The overall decoder is composed of block decoders connected as in Fig. 2, which can be implemented as a pipeline or by feedback.

If a rate b/n convolutional code is used to construct a constituent trellis encoder, we can first use the symbol MAP algorithm to compute the log-likelihood ratio of a symbol $\mathbf{u} = u_1, u_2, \ldots, u_b$ given the observation $\mathbf{y}$ as

$$\lambda(\mathbf{u}) = \log \frac{P(\mathbf{u}|\mathbf{y})}{P(\mathbf{0}|\mathbf{y})}$$

where $\mathbf{0}$ corresponds to the all-zero symbol. Then we obtain the log-likelihood ratios of the jth bit within the symbol by (bit reliability calculation)

$$L(u_j) = \log \frac{\sum_{\mathbf{u}:u_j=1} e^{\lambda(\mathbf{u})}}{\sum_{\mathbf{u}:u_j=0} e^{\lambda(\mathbf{u})}}$$

[3]Note that the components of the $\tilde{\mathbf{L}}_i$'s corresponding to the tail bits, i.e., $\tilde{L}_{ik}$, for $k = N+1, \cdots, N+M_i$, where M_i is the memory of the ith trellis code, are set to zero for all iterations.

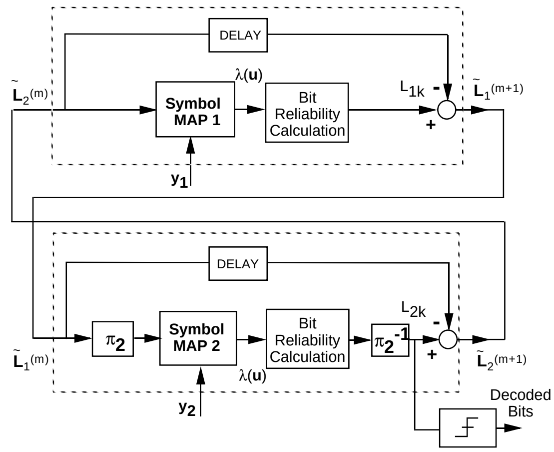

Figure 2: Iterative (Turbo) Decoder Structure for Two Trellis Codes

No. of States	Code Generator			Natural mapping	Gray mapping
	h_0	h_1	h_2	d_{ef}^2	d_{ef}^2
8	13	4	15	4.8	
8	13	17	15		4.8
16	23	16	27	7.2	
16	23	35	27		7.2

Table 3: Rate 2/3 selected constituent codes.

The symbol a priori probabilities required in the symbol MAP algorithm, to be used in the branch transition probability calculation, can be simply found as (Assuming the extrinsic bit reliabilities coming from the other decoder are independent. This is a fair assumption since a bit interleaver and deinterleaver are used in the iterative decoder)

$$P(\mathbf{u} = (u_1, u_2, .., u_b)) = \prod_{j=1}^{b} \frac{e^{u_j \tilde{L}_j}}{1 + e^{\tilde{L}_j}} \quad (5)$$

In this way the iterative (turbo) decoder operates on bits, and bit interleaving, rather than symbol interleaving is used. The bit MAP algorithm for decoding of trellis codes can be also obtained directly, but this issue will not be addressed here and it is deferred to a paper in preparation.

IV. Examples for Parallel Concatenated Trellis Coded Modulation

In this paper we give three examples of application of our proposed method, using 16QAM, 8PSK, and 64QAM constellations and three types of mapping as discussed in Sec. II. The code selection based on maximizing the effective free Euclidean distance of parallel concatenated TCM for a general mapping is still under investigation. In addition to maximizing the d_{ef}, the distance spectrum of the selected codes should also be investigated.

2 bits/sec/Hz PCTCM with 16QAM — The codes we propose have $b = 2$, and employ a 16QAM modulation in connection with two 8-state or two 16-state, rate 2/3 constituent codes. The selected codes for natural and Gray code mapping with the corresponding squared effective free Euclidean distance d_{ef}^2 of PCTCM are given in Table 3 (The average power per dimension is normalized to 1/2)

In our simulation, we selected the 16-state code $h_0 = 23$ $h_1 = 16$ $h_2 = 27$ with natural mapping with two interleavers of size 16384 bits designed according to the procedure described in [4] with parameters S=40 and S=32. The structure of the PCTCM with 16QAM and two clock cycle trellis termination is shown in Fig. 3.

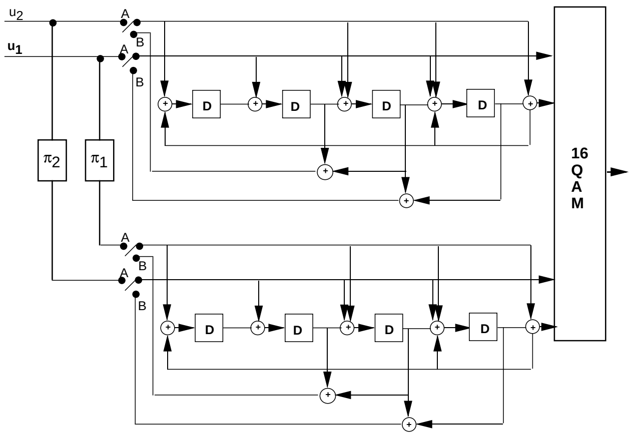

Figure 3: Parallel Concatenated Trellis Coded Modulation, 16QAM, 2 bits/sec/Hz.

To obtain the bit error probability performance, we simulated the iterative decoding structure for two codes as discussed in the previous section. The results are shown in Fig. 4, where 3×10^9 random bits were simulated to measure performance at low BER. As shown by the performance curves, there is an error floor at about BER=10^{-8}. The error floor (change of slope in performance after the breakpoint in the performance curve) can be lowered by increasing d_{ef} and the interleaving size. The distance distribution of a parallel concatenated code plays an important role in minimizing the signal to noise ratio corresponding to the break point.

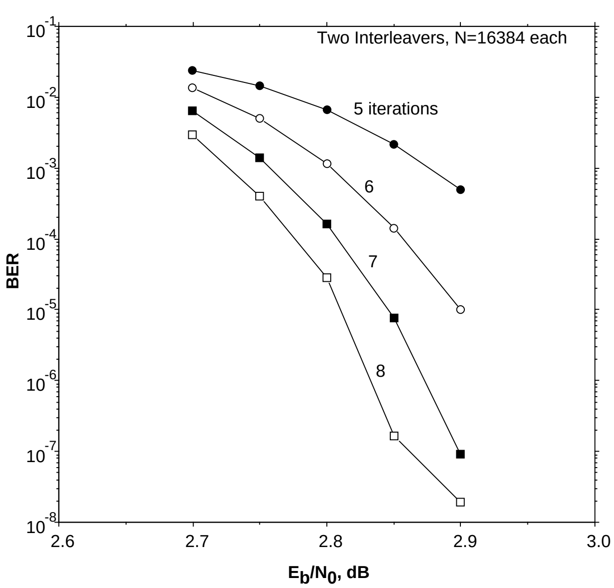

Figure 4: BER Performance of Turbo Trellis Coded Modulation, 16QAM, 2 bits/sec/Hz.

2 bits/sec/Hz PCTCM with 8PSK — The codes we propose have $b = 4$, and employ an 8PSK modulation in connection with two 8-state or two 16-state, rate 4/5 constituent codes. The selected codes for natural, reordered, and Gray code mapping with their corresponding d_{ef}^2 (Using the minimum number of interleavers as shown in parenthesis) are

No. of states	Code Generator h_0	h_1	h_2	h_3	h_4	Natural (4) d_{ef}^2	Reordered (2) d_{ef}^2	Gray (2) d_{ef}^2
8	13	4	6	11	07	6.34	5.17	
8	13	15	17	11	05			5.17
16	23	4	16	37	31	8.34		
16	23	14	16	21	31		6.34	
16	23	35	33	37	31			6.34

Table 4: Rate 4/5 selected constituent codes.

given in Table 4 (unit-norm constellation is assumed)

In our simulation, we selected the 16-state code $h_0 = 23$ $h_1 = 14$ $h_2 = 16$ $h_3 = 21$ $h_4 = 31$ with reordered mapping with four random interleavers, each of size 4096 bits. The structure of these codes with two clock cycle trellis termination is shown in Fig. 5. The bit error probability performance of the selected code is shown in Fig. 6.

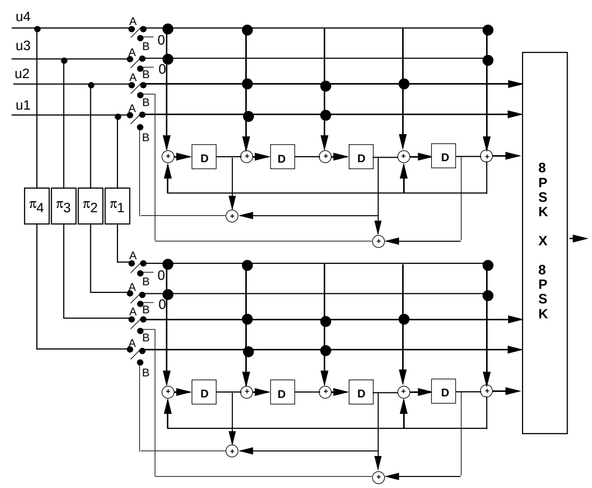

Figure 5: Parallel Concatenated Trellis Coded Modulation, 8PSK, 2 bits/sec/Hz.

4 bits/Sec/Hz PCTCM with 64QAM — The codes proposed for this case have $b = 4$, and employ an 64QAM modulation with two 8-state or two 16-state, rate 4/5 constituent codes (Same as in Table 4). The corresponding squared effective free Euclidean distance d_{ef}^2 is shown in Table 5 (The average power per dimension is normalized to 1/2)

In our simulation we selected again the 16-state code $h_0 = 23$ $h_1 = 14$ $h_2 = 16$ $h_3 = 21$ $h_4 = 31$ with reordered mapping with four random interleavers, each of size 4096 bits. The structure of the PCTCM with 64QAM and two clock cycle trellis termination is shown in Fig. 7. The bit error probability performance of this code is shown in Fig. 8.

For 8PSK and 64QAM 16-state codes, natural mapping did not achieve the best performance at low SNR even though d_{ef} was larger.

V. Conclusions

In this paper we have proposed a new method to construct extremely power and bandwidth efficient parallel concatenated trellis codes with multilevel amplitude/phase modulations. Three significant examples employing rate 2/3, and rate 4/5 constituent codes and 16QAM, 8PSK and 64QAM modulation schemes were described, and their performance was obtained by simulating an iterative decoding algorithm.

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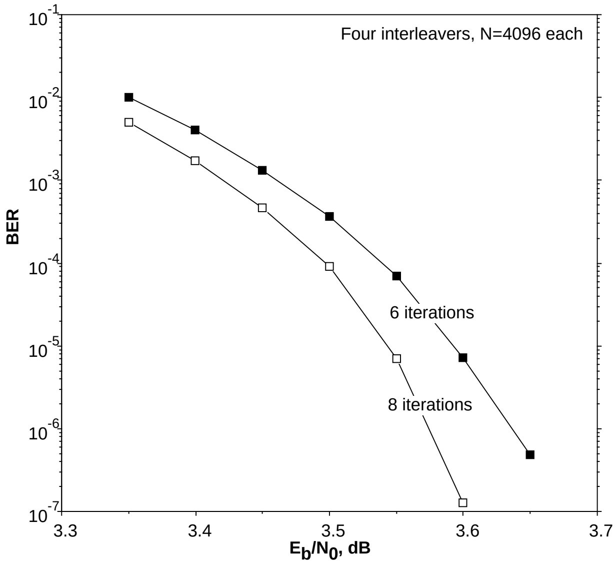

Figure 6: BER Performance of Parallel Concatenated Trellis Coded Modulation, 8PSK, 2 bits/sec/Hz.

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No. of states	Code Generator					Natural (4)	Reordered (2)	Gray (2)
	h_0	h_1	h_2	h_3	h_4	d^2_{ef}	d^2_{ef}	d^2_{ef}
8	13	4	6	11	07	1.14	0.95	
8	13	15	17	11	05			0.95
16	23	4	16	37	31	1.52		
16	23	14	16	21	31		1.14	
16	23	35	33	37	31			1.14

Table 5: Rate 4/5 selected constituent codes.

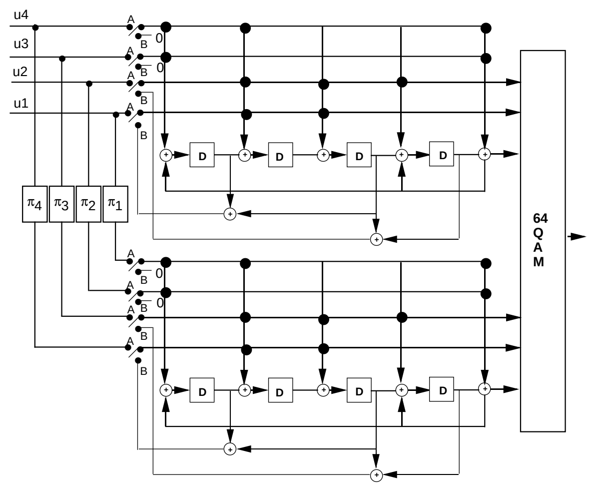

Figure 7: Parallel Concatenated Trellis Coded Modulation, 64QAM, 4 bits/sec/Hz.

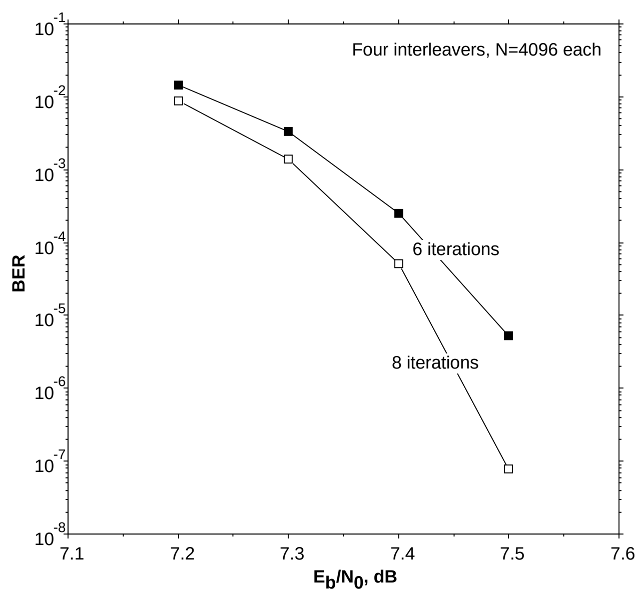

Figure 8: BER Performance of Parallel Concatenated Trellis Coded Modulation, 64QAM, 4 bits/sec/Hz.